Q-R-pre-preserving-conditional[11,40]

$\forall es$:ES, P_1, P_2:(E$\rightarrow$ $\mathbb{P}$), Q_1, R_1, Q_2, R_2:(E$\rightarrow$E$\rightarrow$ $\mathbb{P}$), dcd_P_1:(e:E$\rightarrow$Dec($P_1(e)$)),
f_1:($\{e$:E$|$ $P_1(e)\}$ $\rightarrow$E), f_2:($\{e$:E$|$ $P_2(e)\}$ $\rightarrow$E).
($\forall e$, e':$\{e$:E$|$ $P_1(e)\}$. $\neg(Q_2(f_1(e),f_1(e')))$)
$\Rightarrow$ ($\forall e$, e':$\{e$:E$|$ $P_2(e)\}$. $\neg(Q_1(f_2(e),f_2(e')))$)
$\Rightarrow$ ($\forall e$:$\{e$:E$|$ $P_1(e)\}$, e':$\{e$:E$|$ $P_2(e)\}$.
($\neg(Q_1(f_1(e),f_2(e')))$)
& ($\neg(Q_1(f_2(e'),f_1(e)))$)
& ($\neg(Q_2(f_1(e),f_2(e')))$)
& ($\neg(Q_2(f_2(e'),f_1(e)))$))
$\Rightarrow$ f_1 is Q_1-R_1-pre-preserving on P_1
$\Rightarrow$ f_2 is Q_2-R_2-pre-preserving on P_2
$\Rightarrow$ $[P_1?\ f_1 : f_2]$ is $Q_1 \vee Q_2$-$R_1 \vee R_2$-pre-preserving on $P_1 \vee P_2$